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Con. 7895-12.

(3 Hours)

KR-7034

[ Total Marks : 100

- N. B. : (1) Question No. 1 is compulsory.  
(2) Attempt any four questions out of remaining six questions.  
(3) Figures to the right indicate full marks.

1. (a) Find  $e^A$ , if  $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$  5

(b) Find the orthogonal trajectory of the family of curves  $x^3y - xy^3 = c$ . 5

(c) Integrate the function  $f(z) = x^2 + ixy$  from A (1, 1) to B(2, 4) along the curve  $x = t, y = t^2$ . 5

(d) Consider the following problem - 5

$$\begin{aligned} &\text{Maximise } Z = 2x_1 - 2x_2 + 4x_3 - 5x_4 \\ &\text{Subject to } x_1 + 4x_2 - 2x_3 + 8x_4 \leq 2 \\ &\quad \quad \quad -x_1 + 2x_2 + 3x_3 + 4x_4 \leq 1 \\ &\quad \quad \quad \text{and } x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Determine :

- (i) all basic solutions.
- (ii) all feasible basic solutions.
- (iii) optimal feasible basic solution.

2. (a) If  $f(z) = u + iv$  is analytic and  $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$  find  $f(z)$  6

(b) Compute  $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$ , 7

$$\text{where } A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

(c) Solve the following LPP by Simplex method - 7

$$\begin{aligned} &\text{Minimise } Z = x_1 - 3x_2 + 3x_3 \\ &\text{Subject to } 3x_1 - x_2 + 2x_3 \leq 7 \\ &\quad \quad \quad 2x_1 + 4x_2 \geq -12 \\ &\quad \quad \quad -4x_1 + 3x_2 + 8x_3 \leq 10 \\ &\quad \quad \quad \text{and } x_1, x_2, x_3 \geq 0 \end{aligned}$$

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3. (a) Show that  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  is derogatory and find its minimal polynomial. 6

(b) Solve the following LPP by Big M-method - 7

$$\begin{aligned} \text{Minimise } & Z = 2x_1 + x_2 \\ \text{Subject to } & 3x_1 + x_2 = 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 3 \\ \text{and } & x_1, x_2 \geq 0 \end{aligned}$$

(c) Show that  $f(z) = \sqrt{|xy|}$  is not analytic at the origin although Cauchy-Riemann equations are satisfied at that point. 7

4. (a) Evaluate  $\int_c \frac{z+6}{z^2-4} dz$  where  $c$  is the circle (i)  $|z|=1$ , (ii)  $|z+2|=1$ . 6

(b) Show that the matrix  $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$  is similar to a diagonal matrix. Also 7

find the transforming matrix and the diagonal matrix. 7

(c) Using Duality solve the following LPP -

$$\begin{aligned} \text{Minimise } & Z = 4x_1 + 3x_2 + 6x_3 \\ \text{Subject to } & x_1 + x_3 \geq 2 \\ & x_2 + x_3 \geq 5 \\ \text{and } & x_1, x_2, x_3 \geq 0 \end{aligned}$$

5. (a) Use the dual Simplex method to solve the following LPP - 6

$$\begin{aligned} \text{Maximise } & Z = -3x_1 - 2x_2 \\ \text{Subject to } & x_1 + x_2 \geq 1 \\ & x_1 + x_2 \leq 7 \\ & x_1 + 2x_2 \leq 10 \\ & x_2 \leq 3 \\ \text{and } & x_1, x_2 \geq 0 \end{aligned}$$

(b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$  7

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(c) Find the characteristics equation of the matrix  $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  and verify that 7

it is satisfied by  $A$  and hence, obtain  $A^{-1}$ .

6. (a) Obtain Taylor's or Laurent's series for the function - 6

$$f(z) = \frac{1}{(1+z^2)(z+2)} \text{ for}$$

(i)  $1 < |z| < 2$  and (ii)  $|z| > 2$ .

(b) Obtain the relative maximum or minimum (if any) of the function 7

$$z = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2.$$

(c) Evaluate  $\int_c \frac{z^2}{(z-1)^2(z-2)} dz$  where  $c$  is the circle  $|z| = 2.5$ . 7

7. (a) Find the bilinear transformation which maps the points  $2, i, -2$  onto the points  $1, i, -1$ . 6

(b) Using the method of Lagrangian multipliers solve the following problem 7

$$\text{Optimise } Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

$$\text{Subject to } x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

(c) Verify Laplace's equation for  $u = \left(r + \frac{a^2}{r}\right) \cos \theta$ . Also find  $v$  and  $f(z)$ . 7