(c) Evaluate by using Green's theorem $\int_C (x^2 - y)dx + (2y^2 + x)dy$, where C is the closed region bounded by $y = 4$ and $y = x^2$ (08)

Q4 (a) If $v = 3x^2y + 6xy - y^3$ show that V is Harmonic function. (06)

(b) Find the Eigenvalues of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and Show that matrix satisfies the characteristic equation . (06)

(c) Evaluate (i) $L^{-1}\left\{\frac{1}{s}\tan^{-1}\frac{1}{s}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$ (08)

Q5 (a) Obtain the half range Fourier cosine series expansion for

$f(x) = x(2 - x)$ in $(0,2)$. (06)

(b) Find Eigen value and Eigen Vector Of Matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (06)

(c) Show that $\vec{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$ is conservative Field. Find (i) Scalar potential for $\vec{F}$ (ii) the work done in moving an object in this field From (0,1,-1) to $(\frac{\pi}{2}, -1, 2)$ (08)

Q6 (a) Find the orthogonal trajectory of family of curves given by

$2x - x^3 + 3xy^2 = a$ (06)

(b) Evaluate $\int_0^{\infty} e^{-3t}\, t\sin t\, dt$ (06)

(c) Show that the Matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonaisable. Find the diagonal form D And diagonalizing matrix M. (08)

SEM III Rev-19 ECS QP-10012620

21/11/2022

Time: 3 Hour Max. Marks: 80

Note: 1. Question no. 1 is compulsory.
2. Attempt any **three** questions out of remaining **five** questions.
3. Figures to the right indicate full marks.

Q1 (a) Find $L\left[\frac{(\cos at-\cos bt)}{t}\right]$. (05)

(b) Find the constants k, if $f(z)=r^3\cos k\theta+ir^k\sin 3\theta$ is analytic. (05)

(c) If $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ Find A^{50} (05)

(d) If the vector $\vec{F}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is

is irrotational; find the constants a, b, c. (05)

Q2 (a) Find the analytic function f(z) *in terms of z* whose real part

is $u=\sin x\cosh y$ (06)

(b) Obtain the Fourier series for $f(x)=e^{ax}$ in $(0,2\pi)$ (06)

(c) (i) If $L\{f(t)\}=\frac{1}{s\sqrt{s+1}}$, then find $L\{f(2t)\}$

(ii) Find $L(t^5\cosh t)$ (08)

Q3 (a) Find $L^{-1}\left[\frac{s}{(s^2+4)(s^2+1)}\right]$ by convolution theorem. (06)

(b) Find Fourier expansion of $f(x)=2x-x^2$ in $(0,3)$ (06)